



**ADVANCED GCE**

**MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

**4753/01**

**QUESTION PAPER**

Candidates answer on the printed answer book.

**OCR supplied materials:**

- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Wednesday 19 January 2011**

**Afternoon**

**Duration:** 1 hour 30 minutes

**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER / INVIGILATOR**

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

## Section A (36 marks)

- 1 Given that  $y = \sqrt[3]{1+x^2}$ , find  $\frac{dy}{dx}$ . [4]
- 2 Solve the inequality  $|2x + 1| \geq 4$ . [4]
- 3 The area of a circular stain is growing at a rate of  $1 \text{ mm}^2$  per second. Find the rate of increase of its radius at an instant when its radius is 2 mm. [5]
- 4 Use the triangle in Fig. 4 to prove that  $\sin^2 \theta + \cos^2 \theta = 1$ . For what values of  $\theta$  is this proof valid? [3]

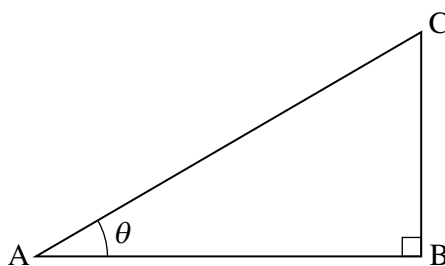


Fig. 4

- 5 (i) On a single set of axes, sketch the curves  $y = e^x - 1$  and  $y = 2e^{-x}$ . [3]
- (ii) Find the exact coordinates of the point of intersection of these curves. [5]
- 6 A curve is defined by the equation  $(x + y)^2 = 4x$ . The point (1, 1) lies on this curve.
- By differentiating implicitly, show that  $\frac{dy}{dx} = \frac{2}{x+y} - 1$ .
- Hence verify that the curve has a stationary point at (1, 1). [4]

3

- 7 Fig. 7 shows the curve  $y = f(x)$ , where  $f(x) = 1 + 2 \arctan x$ ,  $x \in \mathbb{R}$ . The scales on the  $x$ - and  $y$ -axes are the same.

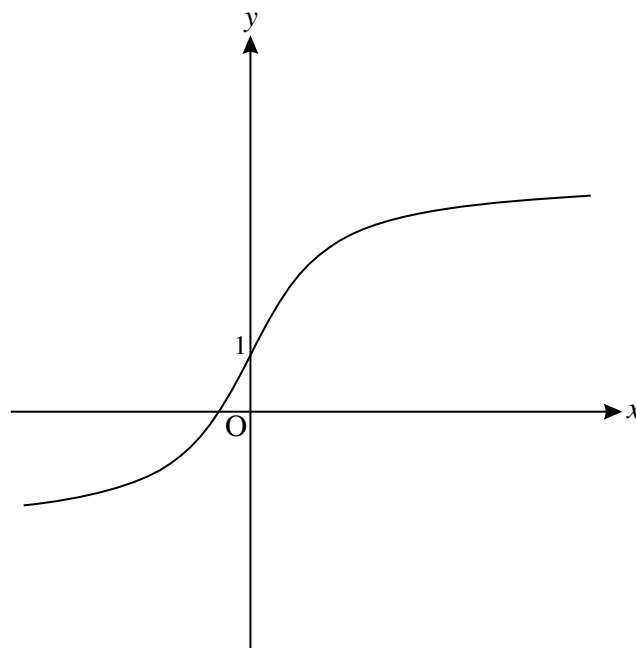


Fig. 7

(i) Find the range of  $f$ , giving your answer in terms of  $\pi$ . [3]

(ii) Find  $f^{-1}(x)$ , and add a sketch of the curve  $y = f^{-1}(x)$  to the copy of Fig. 7. [5]

## Section B (36 Marks)

- 8 (i) Use the substitution  $u = 1 + x$  to show that

$$\int_0^1 \frac{x^3}{1+x} dx = \int_a^b \left( u^2 - 3u + 3 - \frac{1}{u} \right) du,$$

where  $a$  and  $b$  are to be found.

Hence evaluate  $\int_0^1 \frac{x^3}{1+x} dx$ , giving your answer in exact form. [7]

Fig. 8 shows the curve  $y = x^2 \ln(1+x)$ .

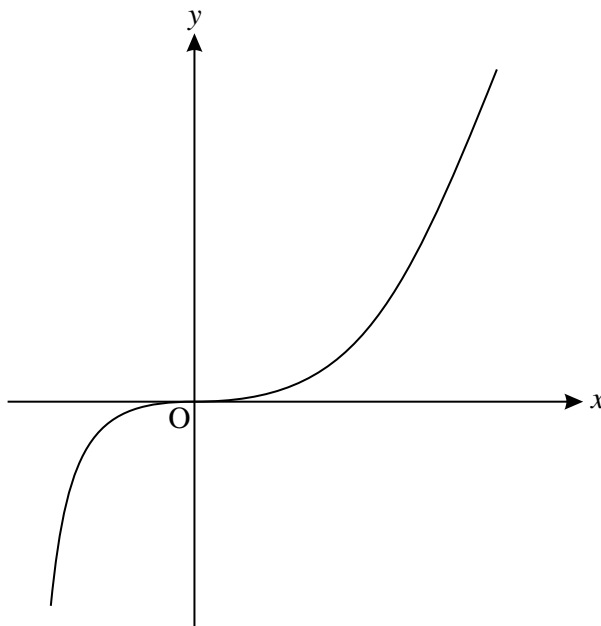


Fig. 8

- (ii) Find  $\frac{dy}{dx}$ .

Verify that the origin is a stationary point of the curve. [5]

- (iii) Using integration by parts, and the result of part (i), find the exact area enclosed by the curve  $y = x^2 \ln(1+x)$ , the  $x$ -axis and the line  $x = 1$ . [6]

5

- 9 Fig. 9 shows the curve  $y = f(x)$ , where  $f(x) = \frac{1}{\cos^2 x}$ ,  $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$ , together with its asymptotes  $x = \frac{1}{2}\pi$  and  $x = -\frac{1}{2}\pi$ .

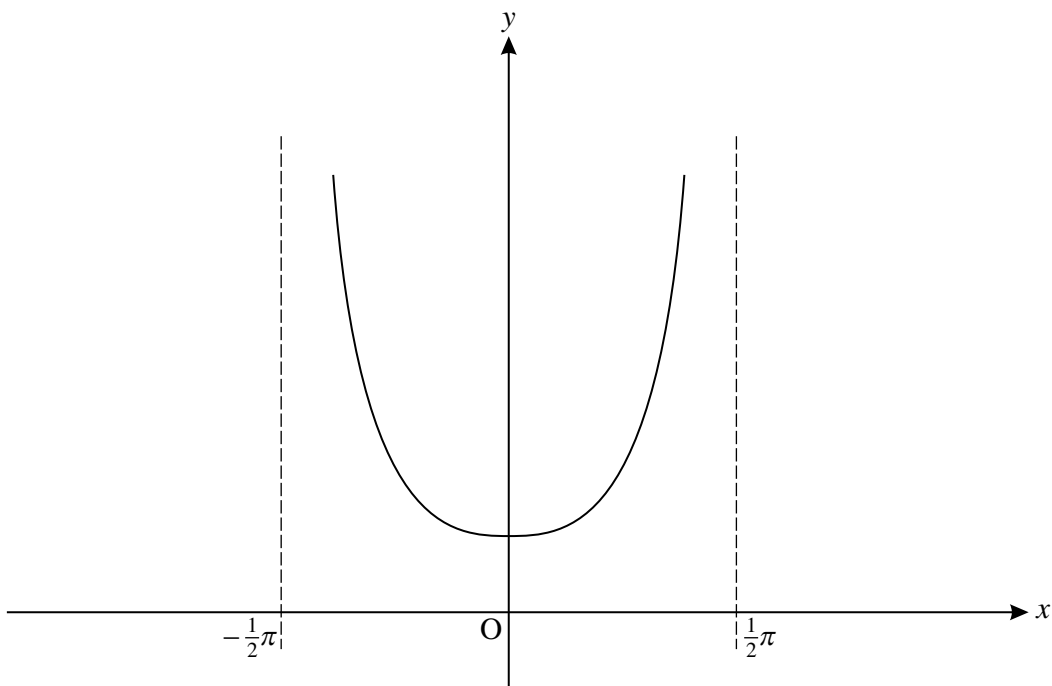


Fig. 9

- (i) Use the quotient rule to show that the derivative of  $\frac{\sin x}{\cos x}$  is  $\frac{1}{\cos^2 x}$ . [3]

- (ii) Find the area bounded by the curve  $y = f(x)$ , the  $x$ -axis, the  $y$ -axis and the line  $x = \frac{1}{4}\pi$ . [3]

The function  $g(x)$  is defined by  $g(x) = \frac{1}{2}f(x + \frac{1}{4}\pi)$ .

- (iii) Verify that the curves  $y = f(x)$  and  $y = g(x)$  cross at  $(0, 1)$ . [3]

- (iv) State a sequence of two transformations such that the curve  $y = f(x)$  is mapped to the curve  $y = g(x)$ . [8]

On the copy of Fig. 9, sketch the curve  $y = g(x)$ , indicating clearly the coordinates of the minimum point and the equations of the asymptotes to the curve. [8]

- (v) Use your result from part (ii) to write down the area bounded by the curve  $y = g(x)$ , the  $x$ -axis, the  $y$ -axis and the line  $x = -\frac{1}{4}\pi$ . [1]

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