OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 Given that $y = \sqrt[3]{1 + x^2}$, find $\frac{dy}{dx}$. **[4]**

2 Solve the inequality $|2x + 1| \geqslant 4$. **[4]**

3 The area of a circular stain is growing at a rate of $1\,\text{mm}^2$ per second. Find the rate of increase of its radius at an instant when its radius is $2\,\text{mm}$. **[5]**

4 Use the triangle in Fig. 4 to prove that $\sin^2\theta + \cos^2\theta = 1$. For what values of θ is this proof valid? **[3]**

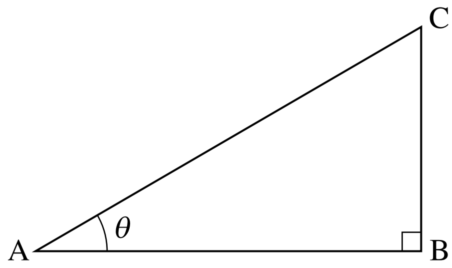

Fig. 4

5 **(i)** On a single set of axes, sketch the curves $y = e^x - 1$ and $y = 2e^{-x}$. **[3]**

(ii) Find the exact coordinates of the point of intersection of these curves. **[5]**

6 A curve is defined by the equation $(x + y)^2 = 4x$. The point $(1, 1)$ lies on this curve.

By differentiating implicitly, show that $\frac{dy}{dx} = \frac{2}{x + y} - 1$.

Hence verify that the curve has a stationary point at $(1, 1)$. **[4]**

7 Fig. 7 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 1 + 2\arctan x$, $x \in \mathbb{R}$. The scales on the x- and y-axes are the same.

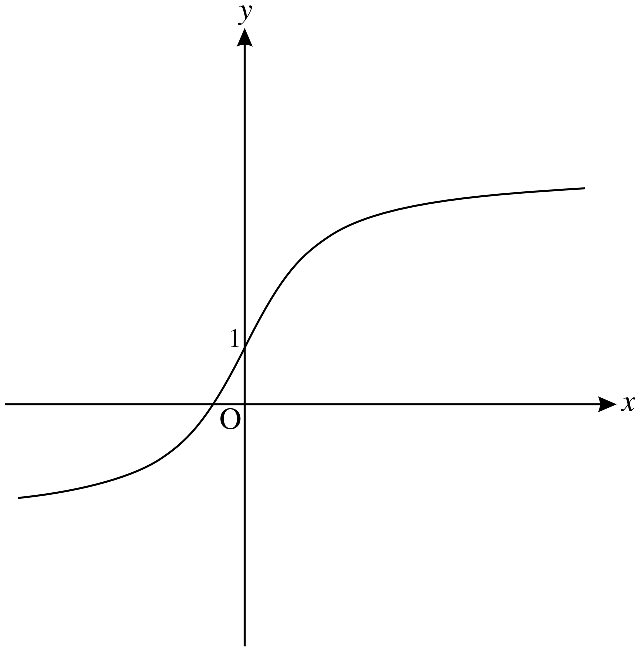

Fig. 7

(i) Find the range of f, giving your answer in terms of π. **[3]**

(ii) Find $\mathrm{f}^{-1}(x)$, and add a sketch of the curve $y = \mathrm{f}^{-1}(x)$ to the copy of Fig. 7. **[5]**

Section B (36 Marks)

8 **(i)** Use the substitution $u = 1 + x$ to show that

$$\int_0^1 \frac{x^3}{1+x}\,dx = \int_a^b \left(u^2 - 3u + 3 - \frac{1}{u}\right) du,$$

where a and b are to be found.

Hence evaluate $\displaystyle\int_0^1 \frac{x^3}{1+x}\,dx$, giving your answer in exact form. **[7]**

Fig. 8 shows the curve $y = x^2 \ln(1 + x)$.

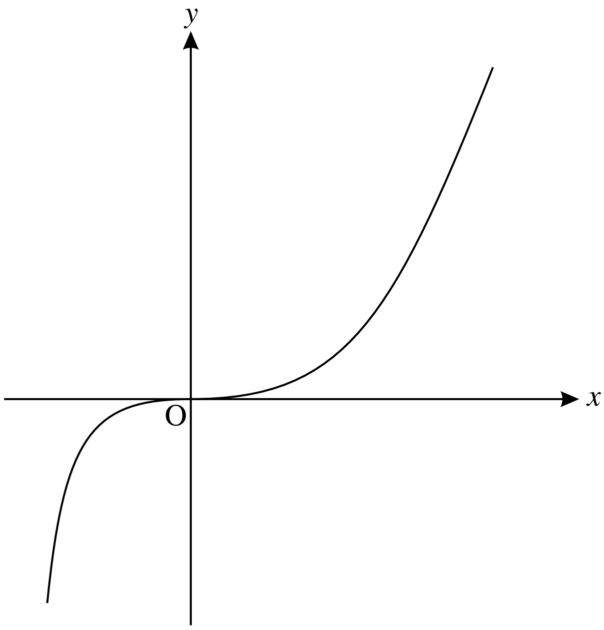

Fig. 8

(ii) Find $\dfrac{dy}{dx}$.

Verify that the origin is a stationary point of the curve. **[5]**

(iii) Using integration by parts, and the result of part **(i)**, find the exact area enclosed by the curve $y = x^2 \ln(1 + x)$, the x-axis and the line $x = 1$. **[6]**

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\cos^2 x}$, $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, together with its asymptotes $x = \frac{1}{2}\pi$ and $x = -\frac{1}{2}\pi$.

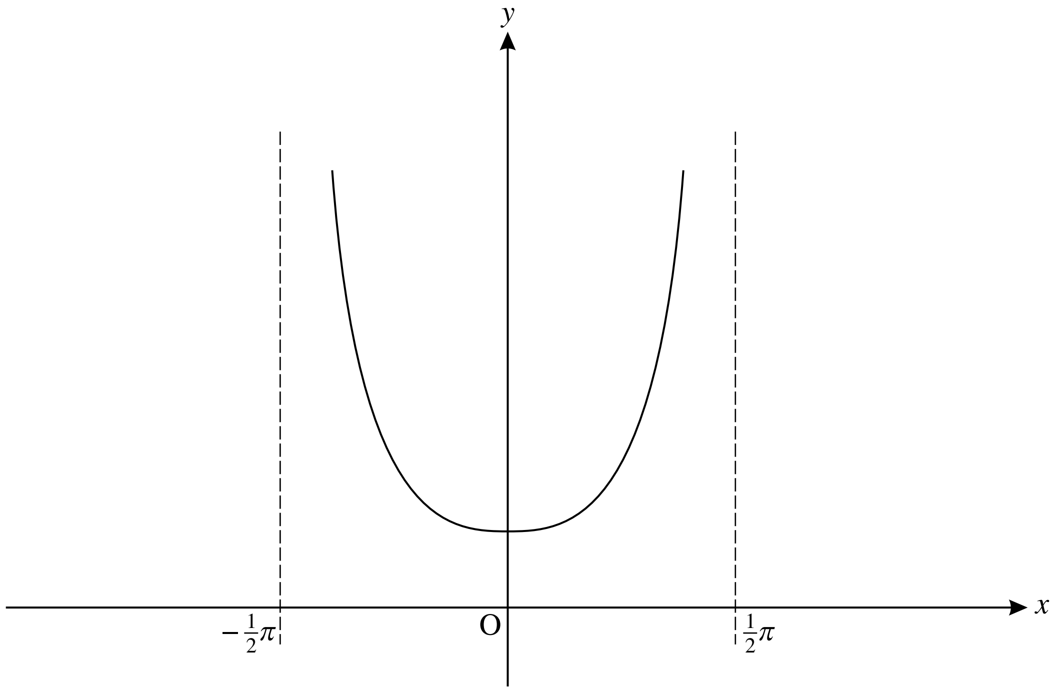

Fig. 9

(i) Use the quotient rule to show that the derivative of $\dfrac{\sin x}{\cos x}$ is $\dfrac{1}{\cos^2 x}$. **[3]**

(ii) Find the area bounded by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \frac{1}{4}\pi$. **[3]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \frac{1}{2}\mathrm{f}\left(x + \frac{1}{4}\pi\right)$.

(iii) Verify that the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ cross at $(0,\ 1)$. **[3]**

(iv) State a sequence of two transformations such that the curve $y = \mathrm{f}(x)$ is mapped to the curve $y = \mathrm{g}(x)$.

On the copy of Fig. 9, sketch the curve $y = \mathrm{g}(x)$, indicating clearly the coordinates of the minimum point and the equations of the asymptotes to the curve. **[8]**

(v) Use your result from part **(ii)** to write down the area bounded by the curve $y = \mathrm{g}(x)$, the x-axis, the y-axis and the line $x = -\frac{1}{4}\pi$. **[1]**

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